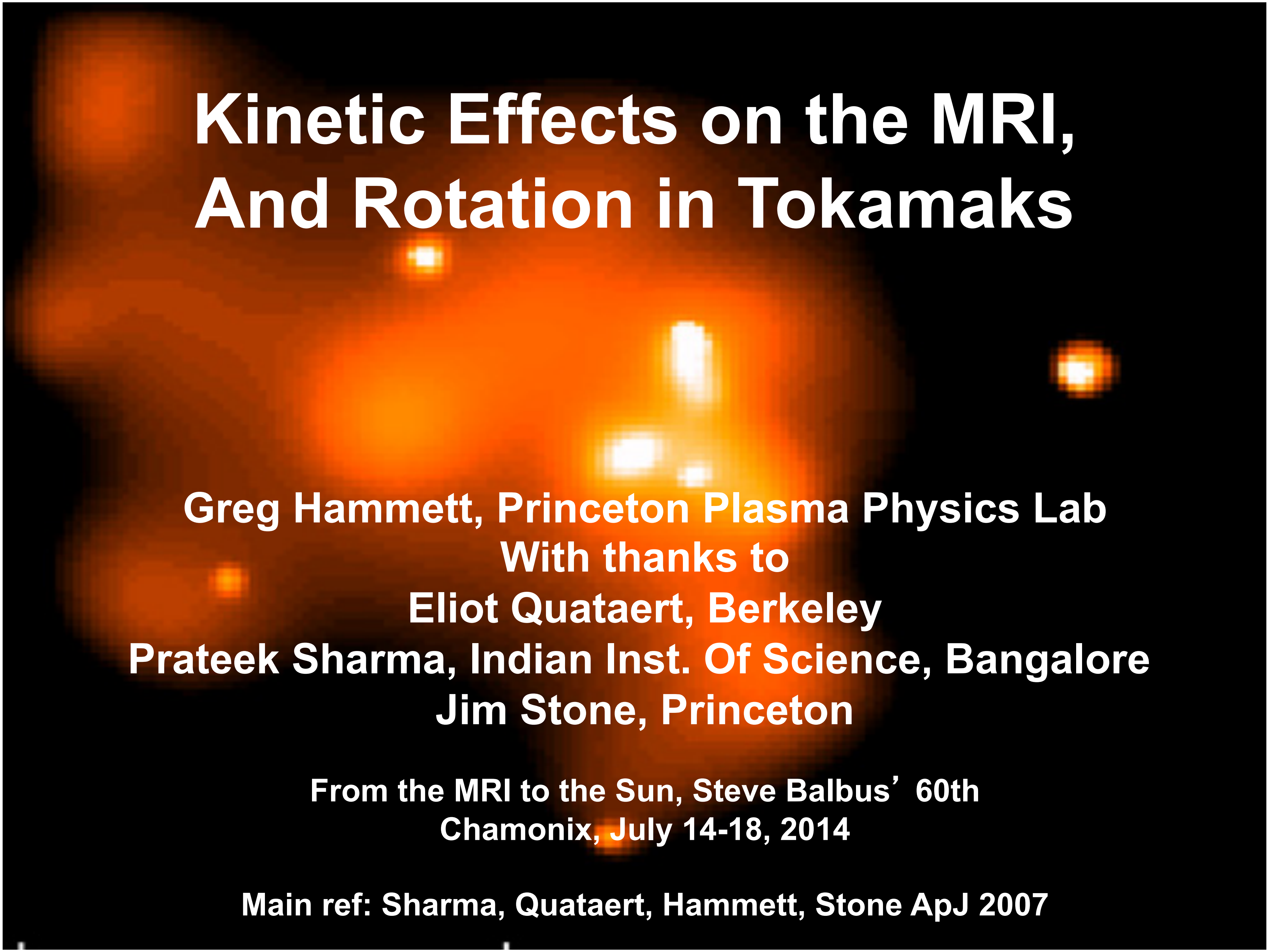
Brief Tasting of Astrophysical Turbulence

Greg Hammett, Princeton Plasma Physics Lab

Things I've learned from: Eliot Quataert (Berkeley), Steve Cowley, Alex Schekochihin, Bill Dorland,
Prateek Sharma (Indian Inst. Of Science, Bangalore)
Greg Howes, Jim Stone, Matt Kunz, Anatoly Spitkovsky,
(and some slides borrowed from Stanislav Boldyrev)

Graduate Summer School, PPPL, Aug. 13-17, 2018



Kinetic Effects on the MRI, And Rotation in Tokamaks

Greg Hammett, Princeton Plasma Physics Lab
With thanks to
Eliot Quataert, Berkeley
Prateek Sharma, Indian Inst. Of Science, Bangalore
Jim Stone, Princeton

From the MRI to the Sun, Steve Balbus' 60th
Chamonix, July 14-18, 2014

Main ref: Sharma, Quataert, Hammett, Stone ApJ 2007

Reading suggestions:

- "MHD Turbulence: A Biased Review", A. Schekochihin (2018, under review) http://www-thphys.physics.ox.ac.uk/research/plasma/JPP/papers17/schekochihin2a.pdf

- Gyrokinetic simulations of the tail of Alfven turbulence, relative electron/ion heating:
  - Quataert & Gruzinov, 1999
  - Howes, Dorland, Cowley Hammett, et al., PRL 2008
  - Howes, TenBarge, Dorland, 2011, other papers
  - Told, Howes, Hammett PRL 2015

- Gyrokinetic phase-space cascades (5D gyrokinetic plasma equivalent of Kolmogorov's $k^{-5/3}$ for 3D fluids): papers by Schekochihin et al., 2008, 2016, ⋯
- Kinetic Effects on MRI turbulence:
  - Sharma, Quataert, & Hammett, (Landau-fluid models) 2000's
  - Matt Kunz (Pegasus hybrid code, fully kinetic ions, role of microinstabilities like firehose & mirror modes)

- MHD-scale astro turbulence, dynamic alignment affects modifying Goldreich-Sridhar:
  - Recent PRLs and papers by Boldyrev, et al., Loureiro et al.,

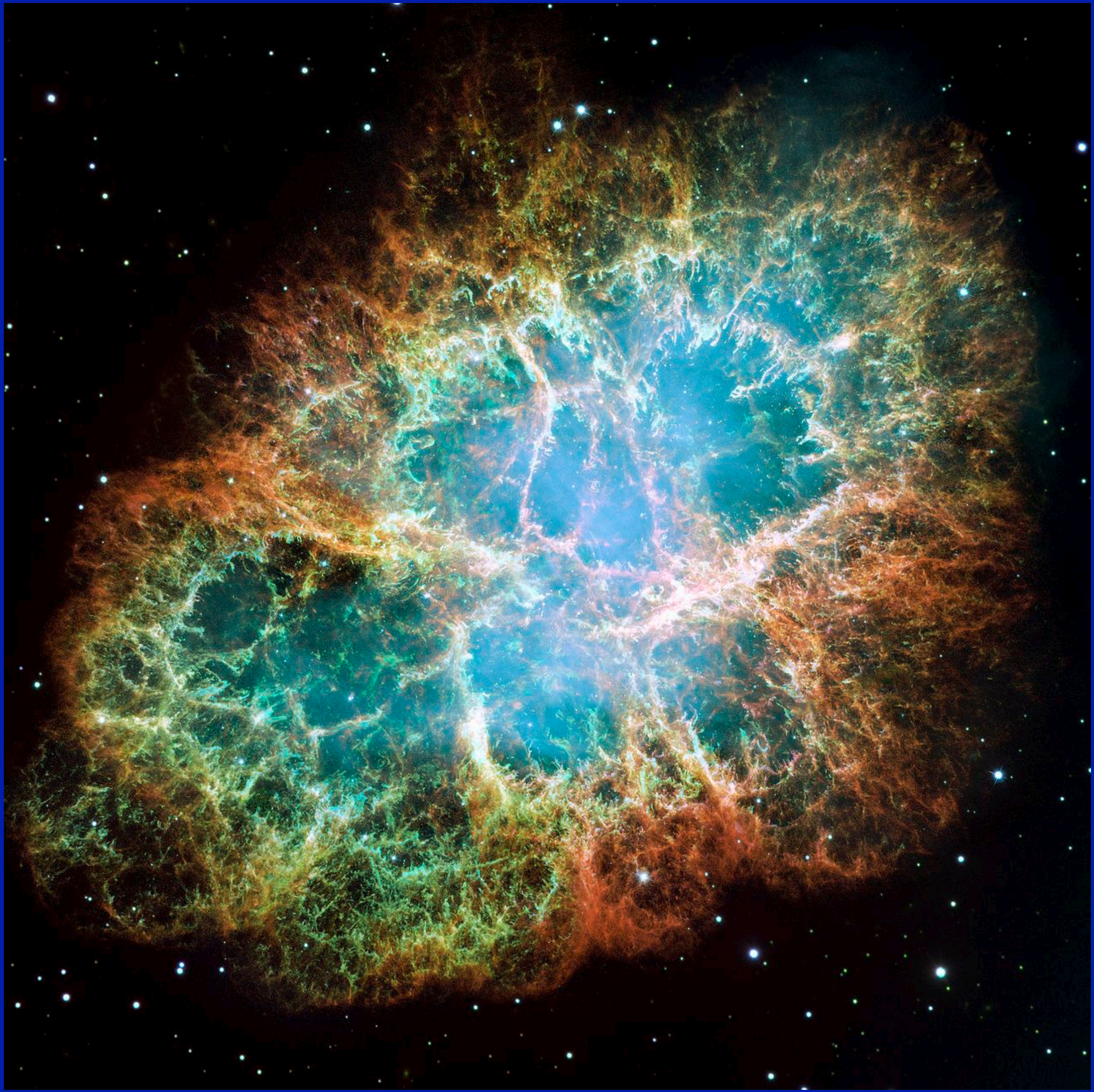
Crab Nebula

Remnant of a supernova explosion in 1054, observed by Chinese astronomers

Filaments: $n \sim 10^9\ m^{-3}$, $T \sim 1$ eV

(Solar wind $n \sim 10^6\ m^{-3}$)

At center is a neutron star, ~ 30 km radius, $M \sim 1.4 - 2\ M_{\odot}$, spinning ~ 30 Hz, emits pulses from radio waves to gamma rays ~ 10 TeV.



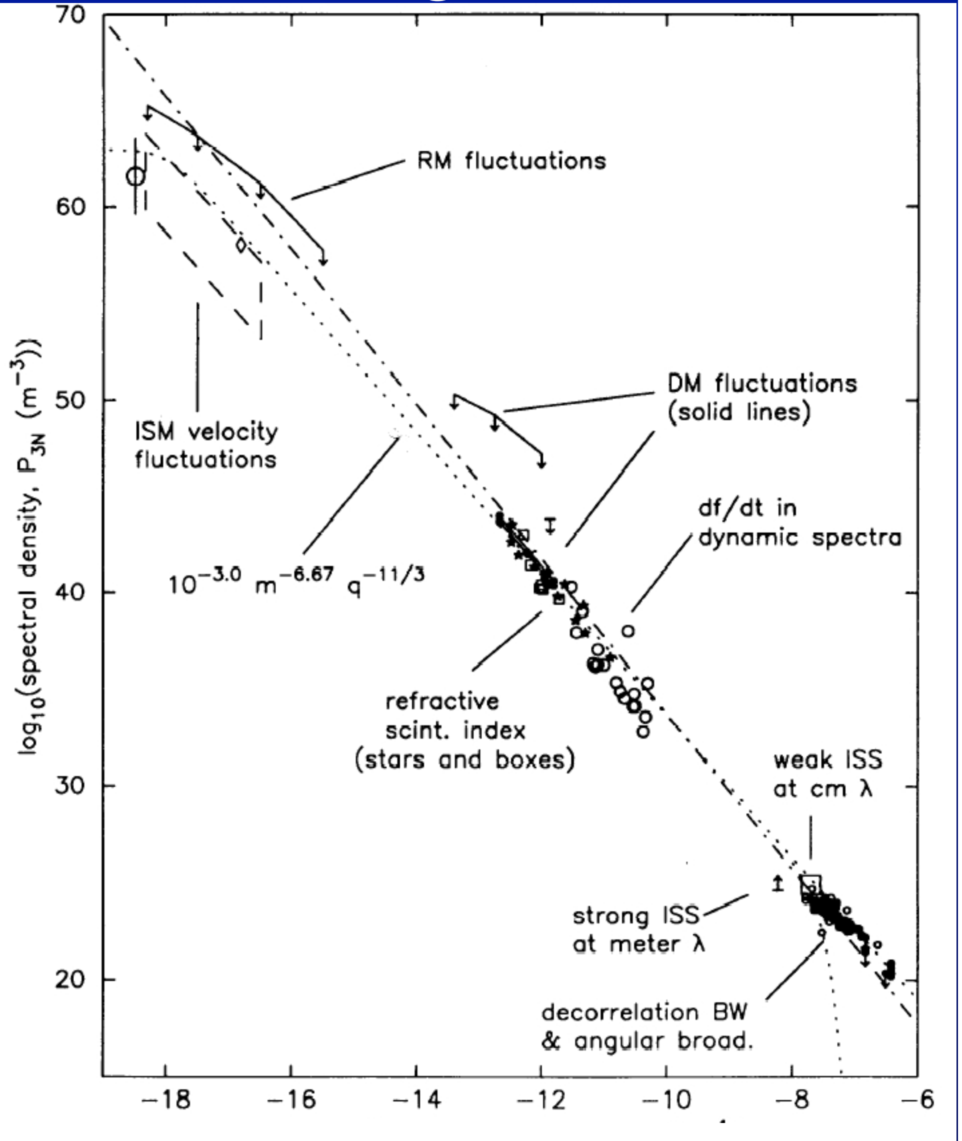
Turbulence in the Interstellar Medium
Power law for 12 orders of magnitude!

Power Spectrum of Electron Density Fluctuations

Density fluctuations change the index of refraction of the plasma & thus modify the propagation of radio waves: "Interstellar scintillation/ scattering"

Consistent with Kolmogorov

$$P_{\rm tot} = \int d^3k\, P_{3\rm N}$$
$$= \int dk\, 4\pi k^2 P_{3\rm N}$$
$$k^2 k^{-11/3} = k^{-5/3}$$



Wavenumber q (m^{-1})

Armstrong, Rickett, Spangler 1995

Why Turbulence Is Important in Astro

- What is the origin of the cosmic magnetic field? How does turbulence drive dynamo generation of magnetic fields?

- How are cosmic ray particles accelerated in turbulent shocks to power law distribution ~$10^9 - 10^{21}$ eV? (injection problem: how accelerated from thermal bulk into energetic tail)

- How are electrons and ions heated? Astronomers primarily measure electrons (visible, x-ray, synchrotron, ...)

- A wide range of astrophysical phenomena are turbulent: supernova explosions (initial explosion, subsequent expansion into space, ...), solar wind, molecular clouds, accretion flows, ...

- How is momentum transported so that matter can collapse into stars, accrete onto black holes? (thousands of papers on turbulence driven by the Balbus & Hawley Magneto-Rotational Instability (MRI)).

There is no single type of "Astrophysical Turbulence". Extreme range of parameters.

- Hot: Relativistic plasmas, "pair plasmas" (positron-electron plasmas)
- Cold: protoplanetary disks, very cold (0.01 − 0.1 eV (50-1500 K) , dusty, but ionization fraction of $\sim 10^{-13}$ sufficient conductivity for MHD turbulence (B & plasma & neutral gas ~ move together)
- beta = p / (B^2/ 8 pi) >> 1 in core of stars,
- beta << 1 in parts of solar corona
- beta ~ 1 in solar wind (B ~ nanotesla)
- $B^2 \sim u^2$ "turbulent equipartition" in many regimes
- $B \sim 10^{11}$ Tesla in Magnetars (type of neutron star)
- Shocks crucial in supernova explosions and cosmic ray acceleration, while incompressible turbulence dominates in many MRI cases
- Magnetic Prandtl number $Pm = D_u / D_B \propto$ viscosity/resistivity <<1 in cold dense plasmas (stars), $\sim 10^{29}$ in hot dilute plasmas (clusters of galaxies)

Plasma & Magnetic Field Effects Often Crucial in Astro Turbulence

- Even very weak magnetic field can suppress thermal conduction, makes conduction very anisotropic
- Even very weak magnetic field has profound effect on shear flow around a gravitationally attracting central object:
  - Stable (linearly & nonlinearly) for neutral gas Navier-Stokes, black holes or other accreting objects would never accrete
  - MRI growth rate ~ orbital frequency, no matter how weak B is., → rapid accretion

- Collisionless plasma shocks (2-stream instabilities, Weibel instability) and microinstabilities (firehose, mirror) often cause bulk heating or acceleration of very energetic tail particles.
- $B^2 \sim u^2$ “turbulent equipartition” in many regimes

Show slides from Boldyrev

Slides borrowed from
https://www.astro.princeton.edu/~kunz/Site/PCTS_files/Diagnosing%20nonlocality,%20anisotropy,%20mesoscale%20structure/Boldyrev.pdf

From a talk by Prof. Stanislav Boldyrev (University of Wisconsin, Madison, and a PPPL Ph.D. alumnus), at the meeting of the Center for Magnetic Self-Organization in Laboratory and Astrophysical Plasmas, in Princeton, 2013.

Boldyrev slides borrowed from
https://www.astro.princeton.edu/~kunz/Site/PCTS_files/Diagnosing%20nonlocality,%20anisotropy,%20mesoscale%20structure/Boldyrev.pdf

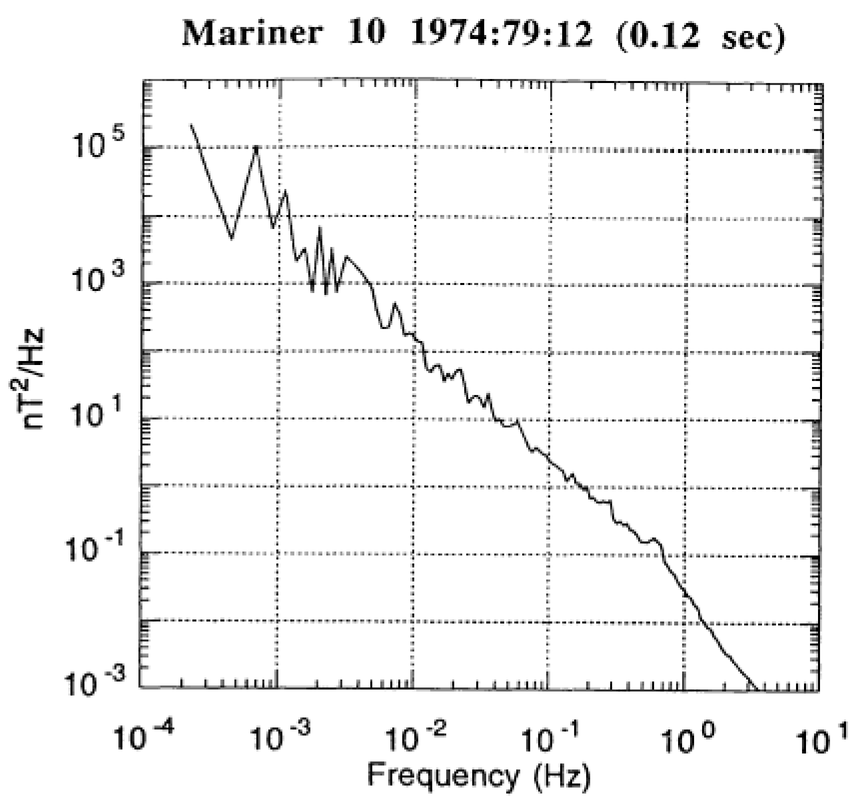
Magnetohydrodynamic Turbulence: solar wind and numerical simulations

Stanislav Boldyrev (UW-Madison)
Jean Carlos Perez (U. New Hampshire)
Fausto Cattaneo (U. Chicago)
Joanne Mason (U. Exeter, UK)
Vladimir Zhdankin (UW-Madison)
Konstantinos Horaites (UW-Madison)
Qian Xia (UW-Madison)

Center for Magnetic Self-Organization in Laboratory and Astrophysical Plasmas
Princeton, April 10, 2013

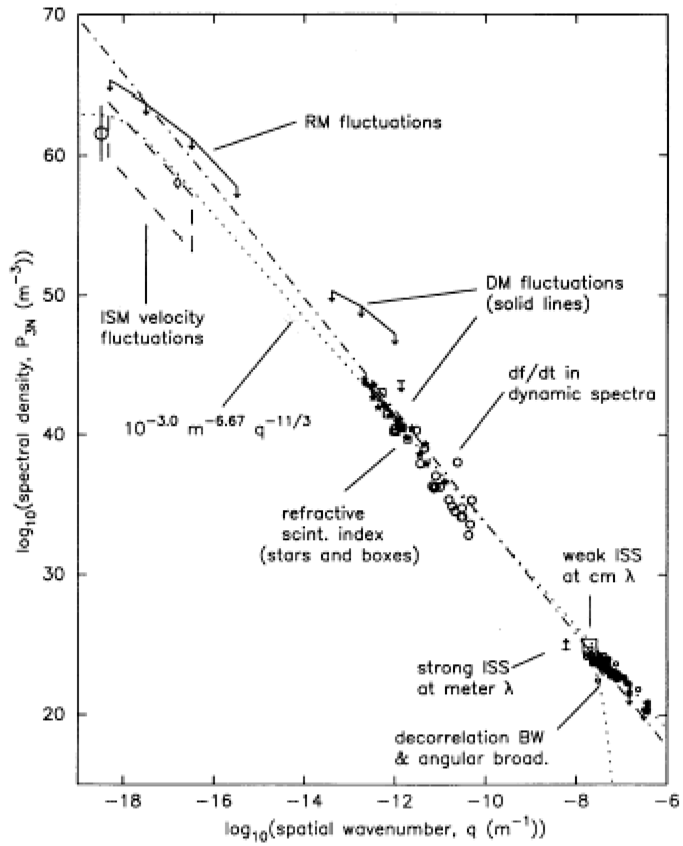
Magnetic turbulence in nature

energy spectra



Solar wind

[Goldstein, Roberts, Matthaeus (1995)]



ISM

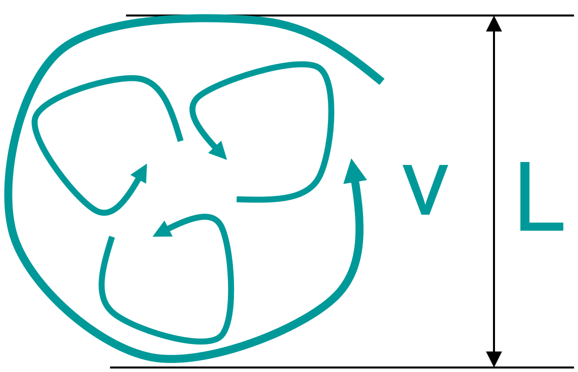
[Armstrong, Rickett, Spangler (1995)]

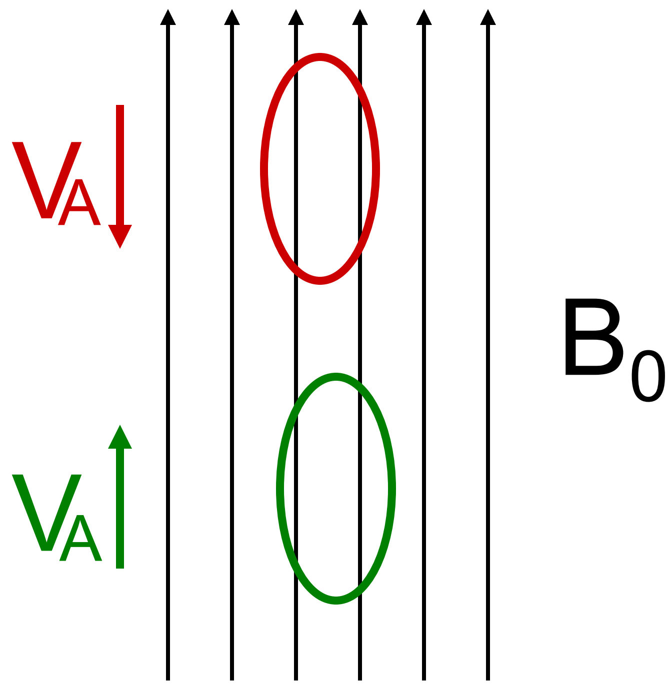
slide from Boldyrev, 2013

Nature of Magnetohydrodynamic (MHD) turbulence

HD turbulence:
interaction of eddies

MHD turbulence:
interaction of wave packets
moving with Alfven velocities

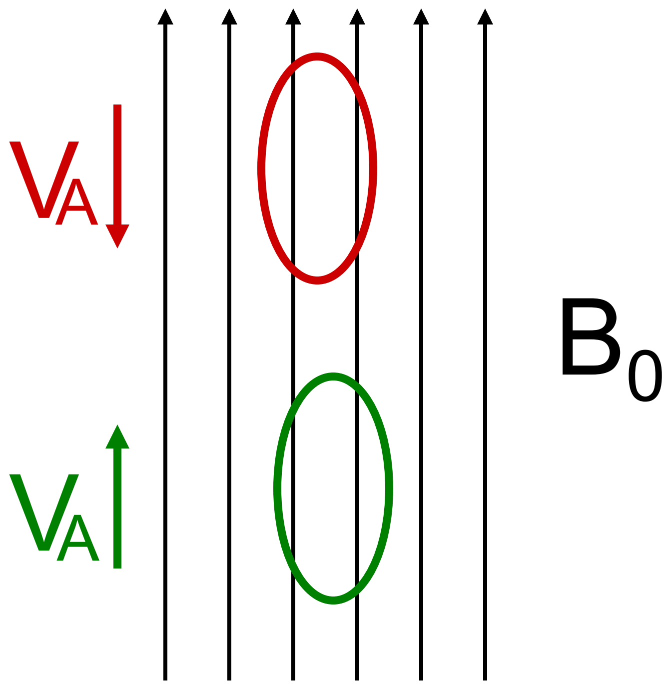




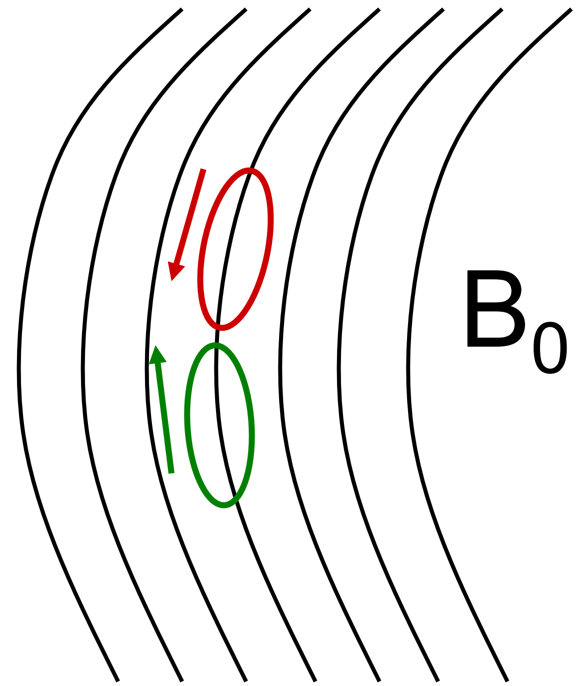
$$\mathbf{v}_A = \mathbf{B}_0/\sqrt{4\pi\rho_0}$$

slide from Boldyrev, 2013

Guide field in MHD turbulence



B_0 imposed by external sources



B_0 created by large-scale eddies

slide from Boldyrev, 2013

Magnetohydrodynamic (MHD) equations

$$\partial_t \mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\frac{1}{\rho_0}\nabla p + \frac{1}{4\pi\rho_0}(\mathbf{B}\cdot\nabla)\mathbf{B} + \nu\nabla^2\mathbf{v}$$

$$\partial_t \mathbf{B} = \nabla\times[\mathbf{v}\times\mathbf{B}] + \eta\nabla^2\mathbf{B}$$

(incompressible limit)

$$\vec{j}\times\vec{B} \propto (\nabla\times\vec{B})\times\vec{B} = \vec{B}\cdot\nabla\vec{B} - \frac{1}{2}\nabla B^2$$

Separate the uniform magnetic field: $\mathbf{B} = \mathbf{B}_0 + \mathbf{b}$

Introduce the Elsasser variables: $\mathbf{z}^{\pm} = \mathbf{v} \pm \frac{1}{\sqrt{4\pi\rho_0}}\mathbf{b}$

Then the equations take a symmetric form:

$$\partial_t \mathbf{z}^+ - (\mathbf{v}_A\cdot\nabla)\mathbf{z}^+ + (\mathbf{z}^-\cdot\nabla)\mathbf{z}^+ = -\nabla P$$

$$\partial_t \mathbf{z}^- + (\mathbf{v}_A\cdot\nabla)\mathbf{z}^- + (\mathbf{z}^+\cdot\nabla)\mathbf{z}^- = -\nabla P$$

With the Alfven velocity $\mathbf{v}_A = \mathbf{B}_0/\sqrt{4\pi\rho_0}$

The uniform magnetic field mediates small-scale turbulence

slide from Boldyrev, 2013

MHD turbulence: Alfvenic cascade

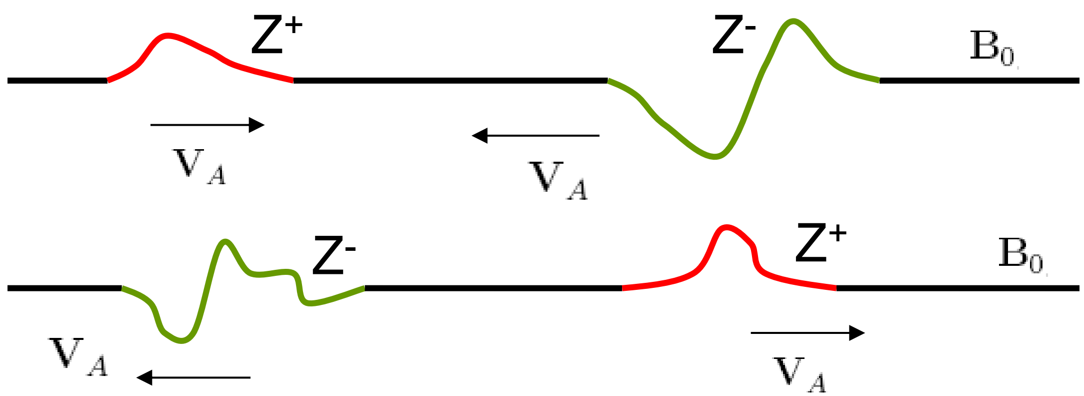
$$\partial \mathbf{z}^{\pm} \mp (\mathbf{v}_A \cdot \nabla)\mathbf{z}^{\pm} + (\mathbf{z}^{\mp} \cdot \nabla)\mathbf{z}^{\pm} = -\nabla P + \frac{1}{Re}\nabla^2 \mathbf{z}^{\pm} + \mathbf{f}^{\pm}$$

Ideal system conserves the Elsasser energies

$$E^{+} = \int (\mathbf{z}^{+})^2 \, d^3x \qquad E^{-} = \int (\mathbf{z}^{-})^2 \, d^3x$$

$$\equiv$$

$$E = \frac{1}{2}\int (v^2 + b^2) d^3x \qquad H^C = \int (\mathbf{v}\cdot\mathbf{b}) d^3x$$



$E^+ \sim E^-$: balanced case.

$E^+ \gg E^-$: imbalanced case

$$H^C = \int (\mathbf{v}\cdot\mathbf{b}) d^3x = \frac{1}{4}(E^+ - E^-) \neq 0$$

slide from Boldyrev, 2013

Strength of interaction in MHD turbulence

$$\partial \mathbf{z}^{\pm} \mp (\mathbf{v}_A \cdot \nabla)\mathbf{z}^{\pm} + (\mathbf{z}^{\mp} \cdot \nabla)\mathbf{z}^{\pm} = -\nabla P + \frac{1}{Re}\nabla^2 \mathbf{z}^{\pm} + \mathbf{f}^{\pm}$$

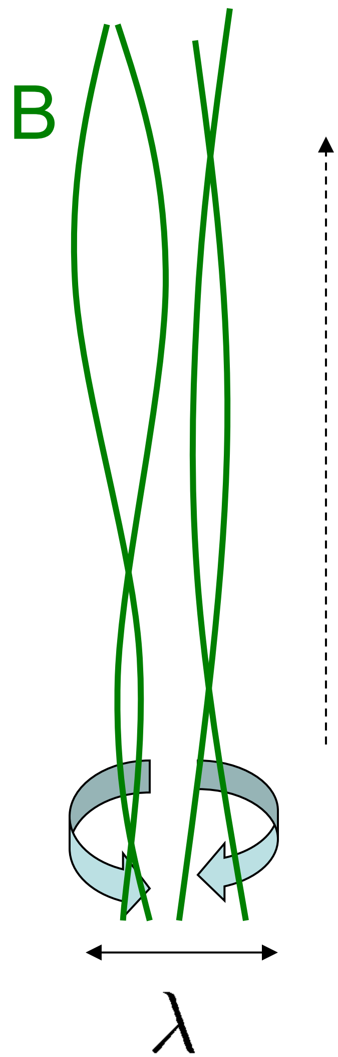
$$\underbrace{(\mathbf{v}_A \cdot \nabla)\mathbf{z}^{\pm}}_{(k_{||} v_A) z^{\pm}} \qquad \underbrace{(\mathbf{z}^{\mp} \cdot \nabla)\mathbf{z}^{\pm}}_{(k_{\perp} z^{\mp}) z^{\pm}}$$

When $k_{||} v_A \gg k_{\perp} z^{\mp}$ turbulence is weak

When $k_{||} v_A \sim k_{\perp} z^{\mp}$ turbulence is strong

slide from Boldyrev, 2013

Goldreich-Sridhar theory: critical balance



$$\partial_t \mathbf{z}^+ - (\mathbf{v}_A \cdot \nabla)\mathbf{z}^+ + (\mathbf{z}^- \cdot \nabla)\mathbf{z}^+ = -\nabla P$$
$$\partial_t \mathbf{z}^- + (\mathbf{v}_A \cdot \nabla)\mathbf{z}^- + (\mathbf{z}^+ \cdot \nabla)\mathbf{z}^- = -\nabla P$$

$$V_A/l \sim \delta b_\lambda/\lambda$$

$$l \sim V_A \tau_\lambda \quad \Rightarrow \quad l \propto \lambda^{2/3}$$

Causality — GS Critical balance

$$\tau_\lambda \sim \lambda/\delta v_\lambda$$

Correlation time of fluctuations, or eddy turnover time

critical balance of strong turbulence
is a consequence of causality

SB (2005)

slide from Boldyrev, 2013

Critical Balance

Parallel linear frequency $\sim$ Nonlinear perpendicular scattering rate

$$\vec{v}_A \cdot \nabla \sim \vec{v}_\perp \cdot \nabla_\perp$$

nonlinear interactions between modes

with comparable k dominate:

$$\vec{v}_{\perp(k)} \sim \left(\int_{k/\sqrt{2}}^{k\sqrt{2}} dk' \, E(k') \right)^{1/2}$$

$$\sim (\Delta k \, E(k))^{1/2}$$

$$\sim k^{1/2} E^{1/2}(k)$$

so:

$$v_A k_{||} \sim \vec{v}_{(\perp k)} \cdot k_\perp$$

$$v_A k_{||} \sim k^{3/2} (k^{-5/3})^{1/2}$$

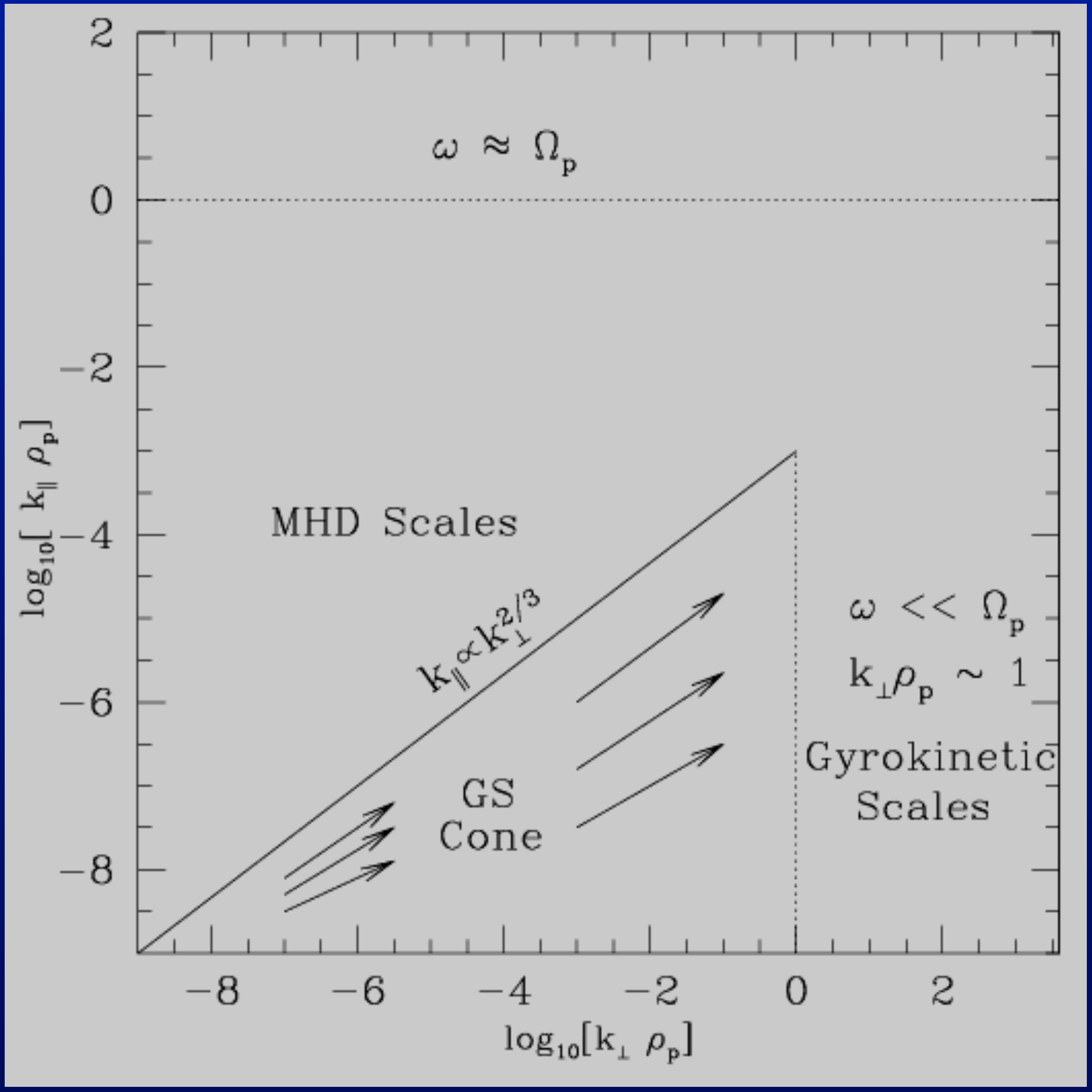
$$v_A k_{||} \sim k_\perp^{2/3}$$

Concept originated with Goldreich-Sridhar, 1994, for astrophysical Alfven wave turbulence.

Barnes, Parra, Schekochihin, PRL 2011 apply to electrostatic ITG turbulence in tokamaks.

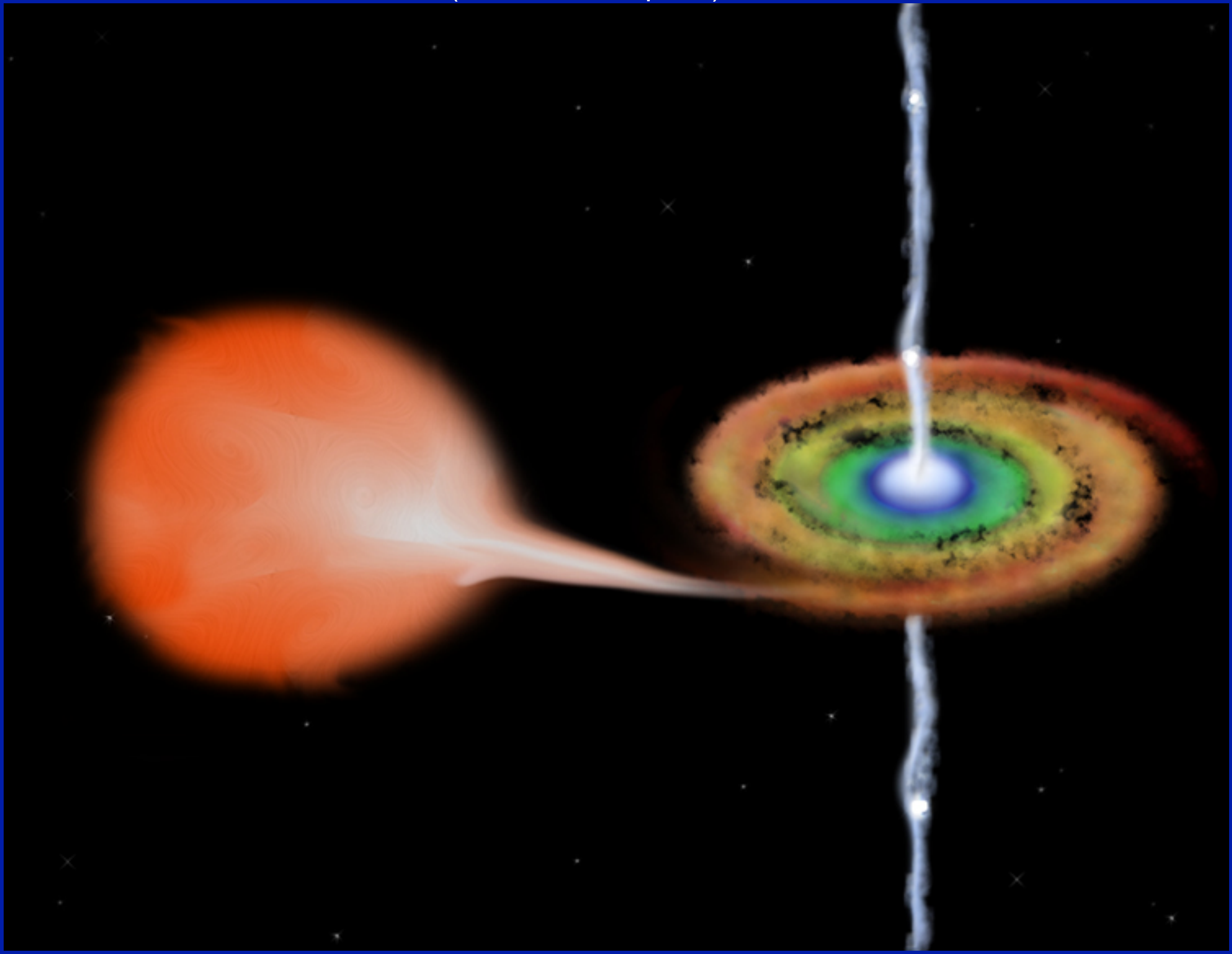
(back to Hammett slides)

MHD ⇒ Gyrokinetics



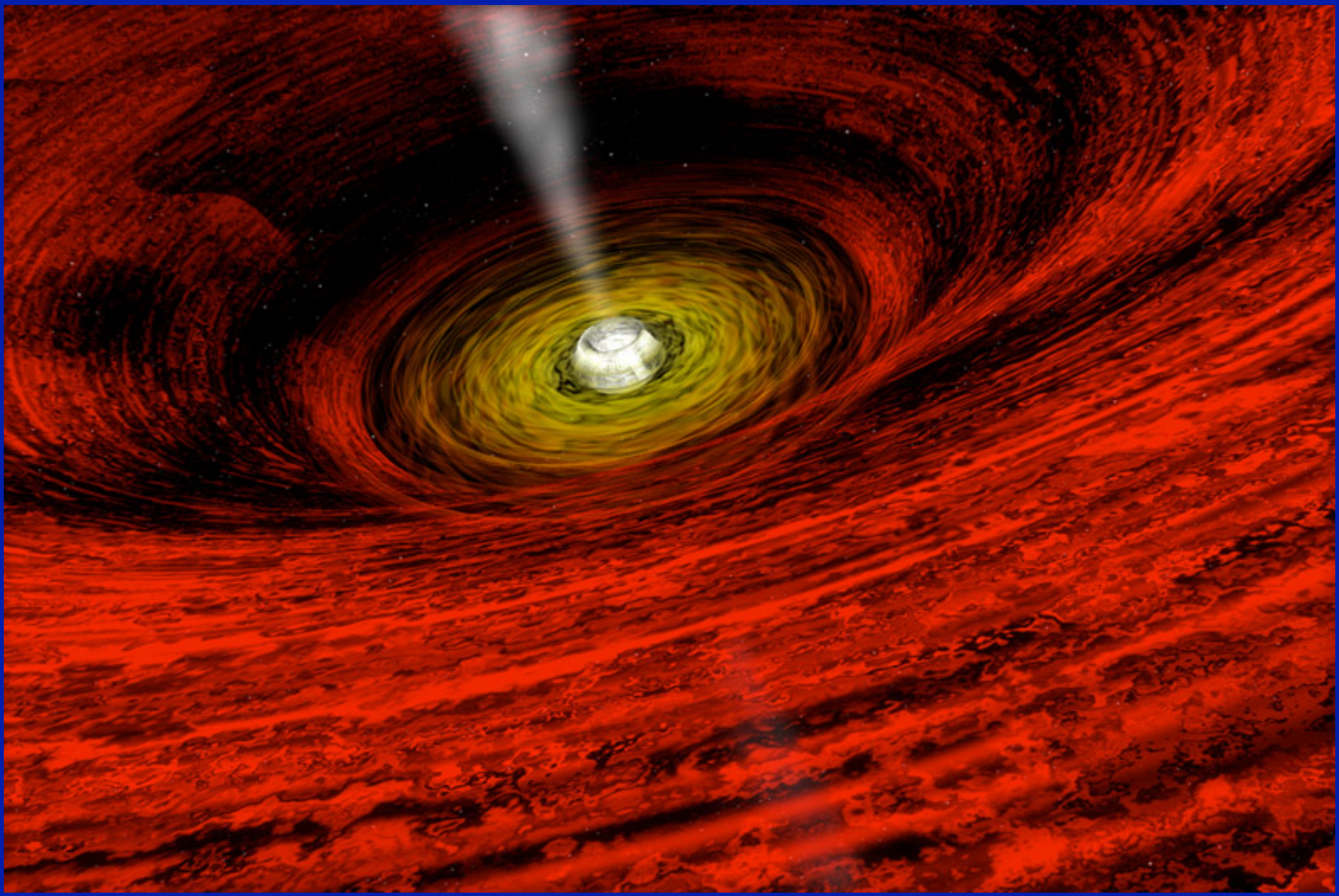
Star orbiting black hole & feeding accretion disk

(artist's conception)



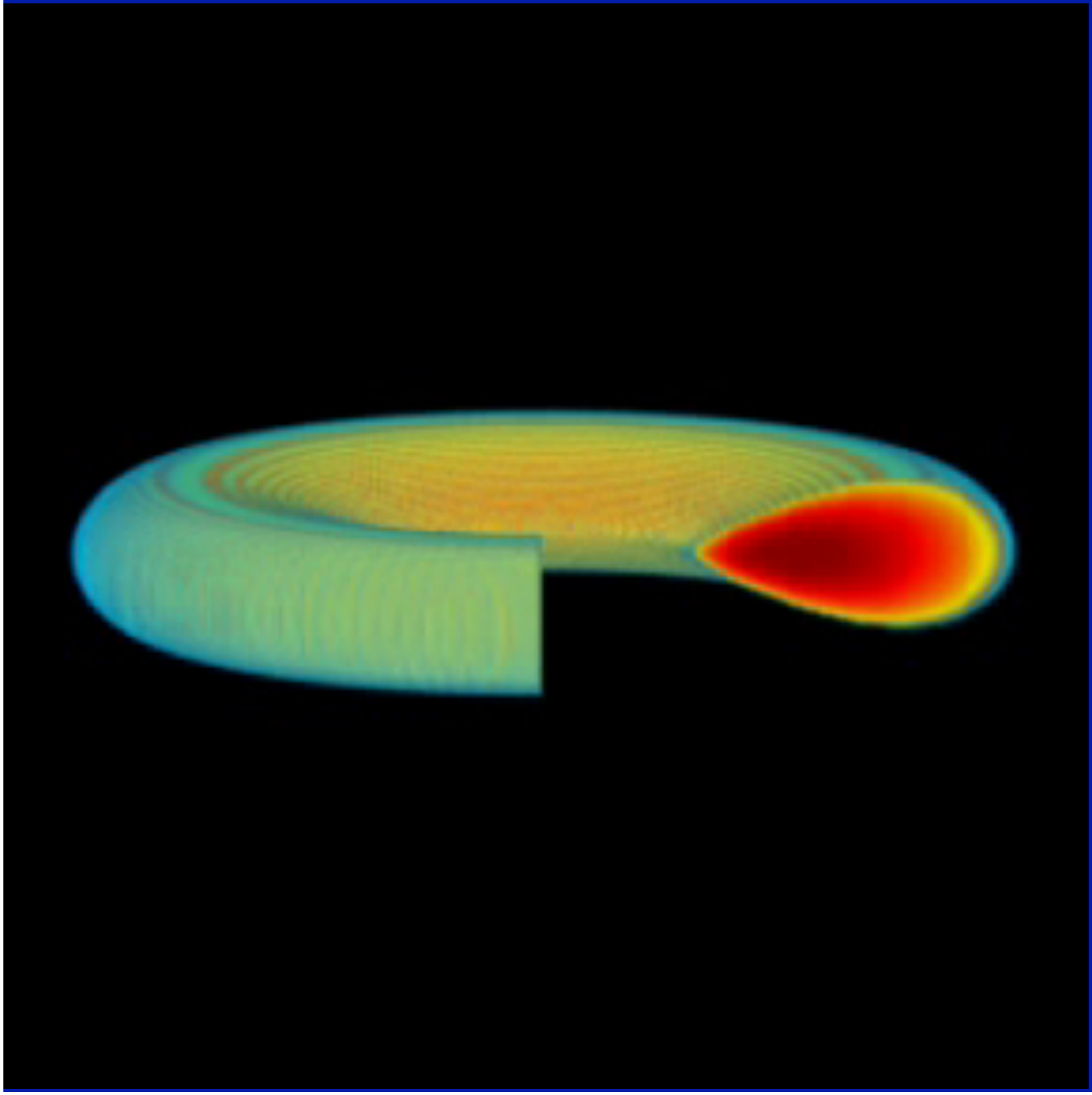
NASA/CXC/SAO A.Hobart   http://chandra.harvard.edu/resources/illustrations.html

Black Hole Neighborhood. (artist’ s conception)



NASA/CXC/SAO A.Hobart http://chandra.harvard.edu/resources/illustrations.html

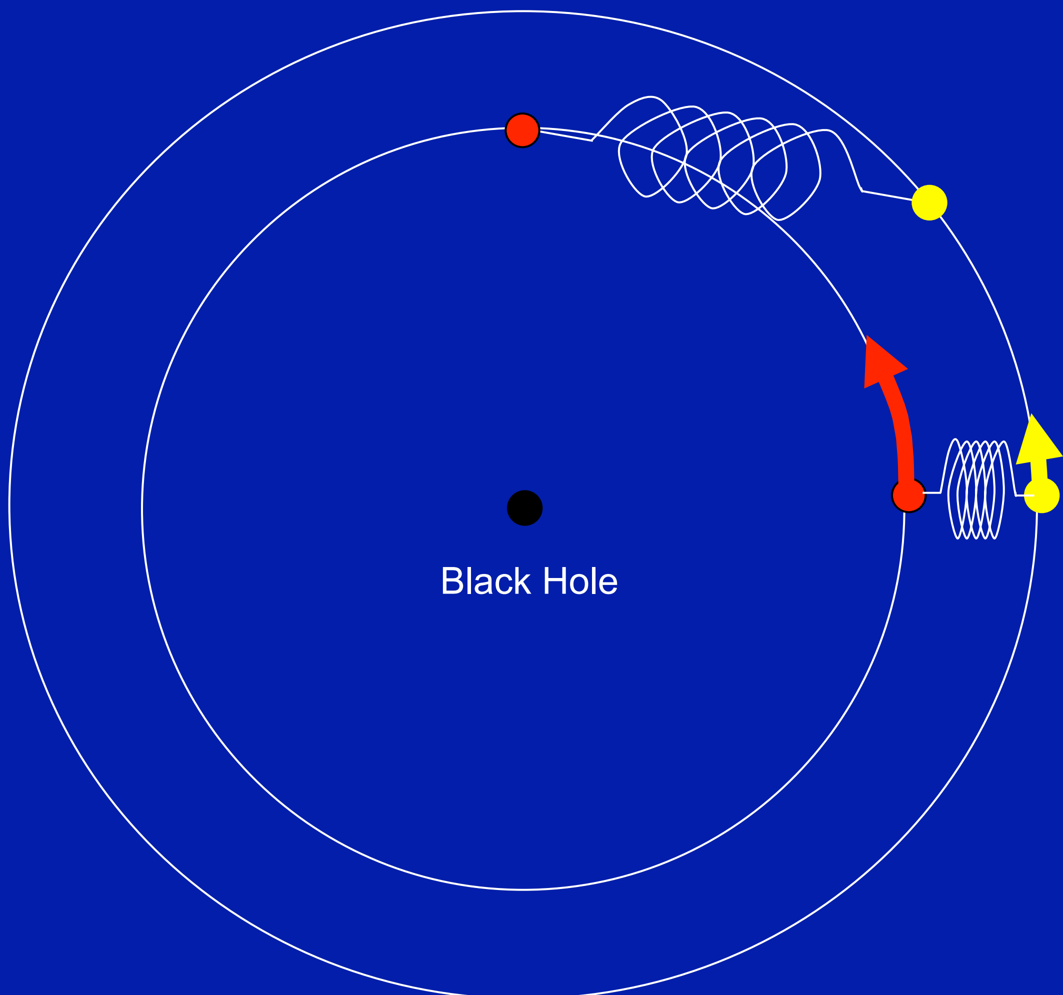
A 3-D Global MHD Simulation



Simulation by Hawley et al.
http://astsun.astro.virginia.edu/~jh8h/

MHD simulations of MRI turbulence very successful. Need to study it in collisionless regime applicable to Sgr A*

Magneto-Rotational Instability explains how accretion disks accrete (Balbus & Hawley, 1991)

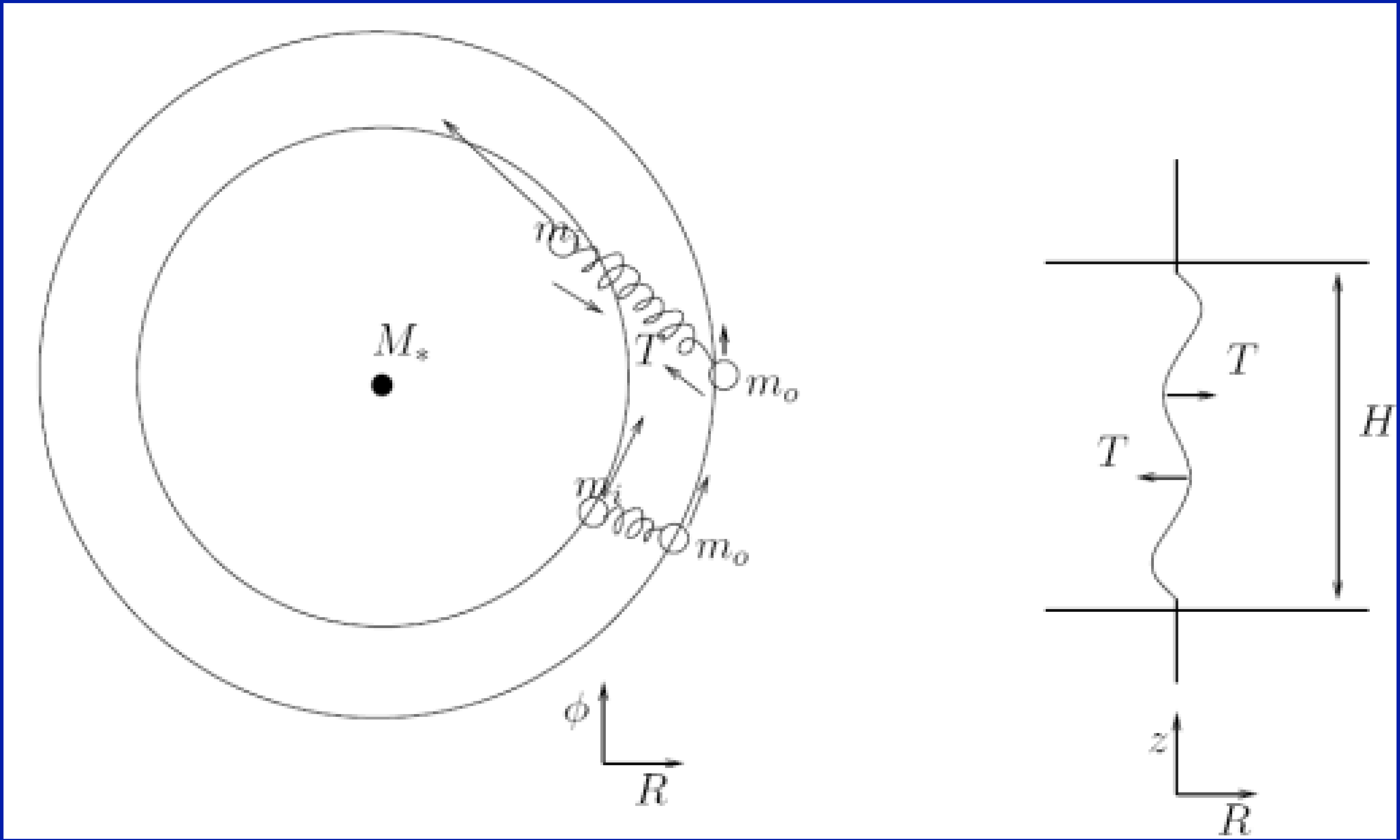


Inner particle orbits faster,
Spring stretches out
Spring force slows inner particle
and accelerates outer particle
Causing inner particle to fall in
and outer particle to go out
Exponentially amplified.

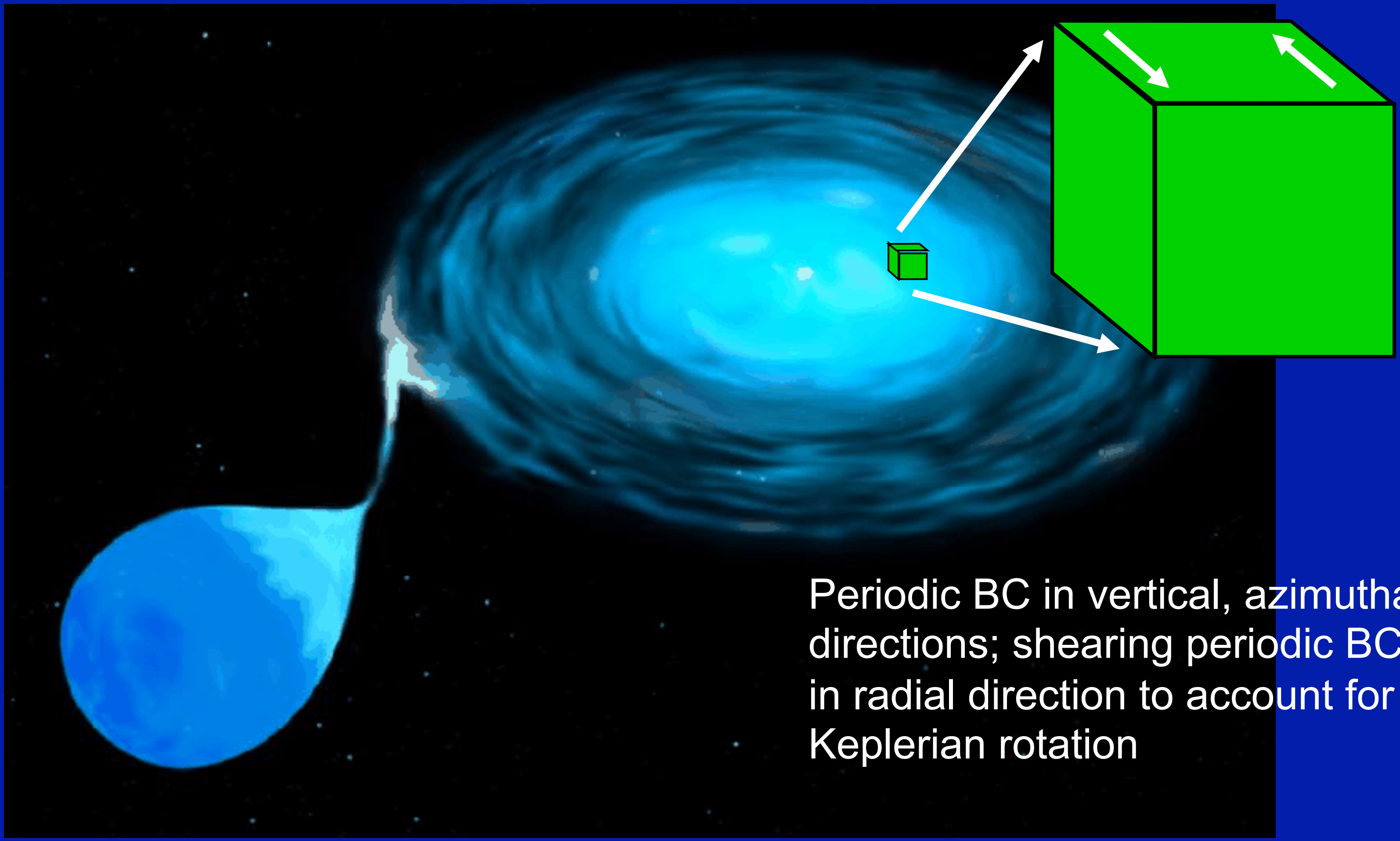
Magnetic fields
Are like springs

spring analogy by Toomre

Side view: magnetic field stretching acts like springs
& transfer angular momentum

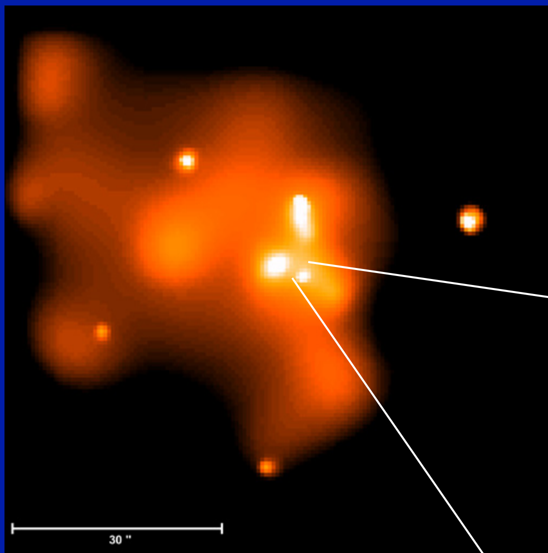


Shearing Box Simulations



Periodic BC in vertical, azimuthal directions; shearing periodic BC in radial direction to account for Keplerian rotation

Galactic Center BH



Chandra

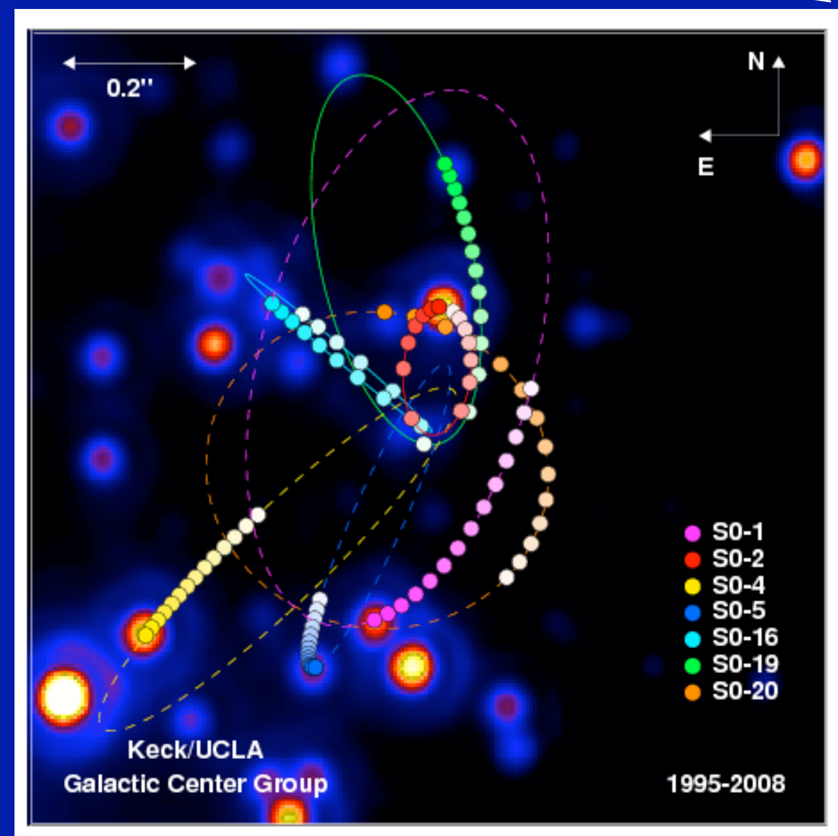
$3.6 \times 10^6\ M_o$ black hole

Bondi radius ~ 0.07 pc (2''),
n~100/cc, T~1-2 keV

$\dot{M} \sim 10^{-5}\ M_o$ /yr by stellar outflows

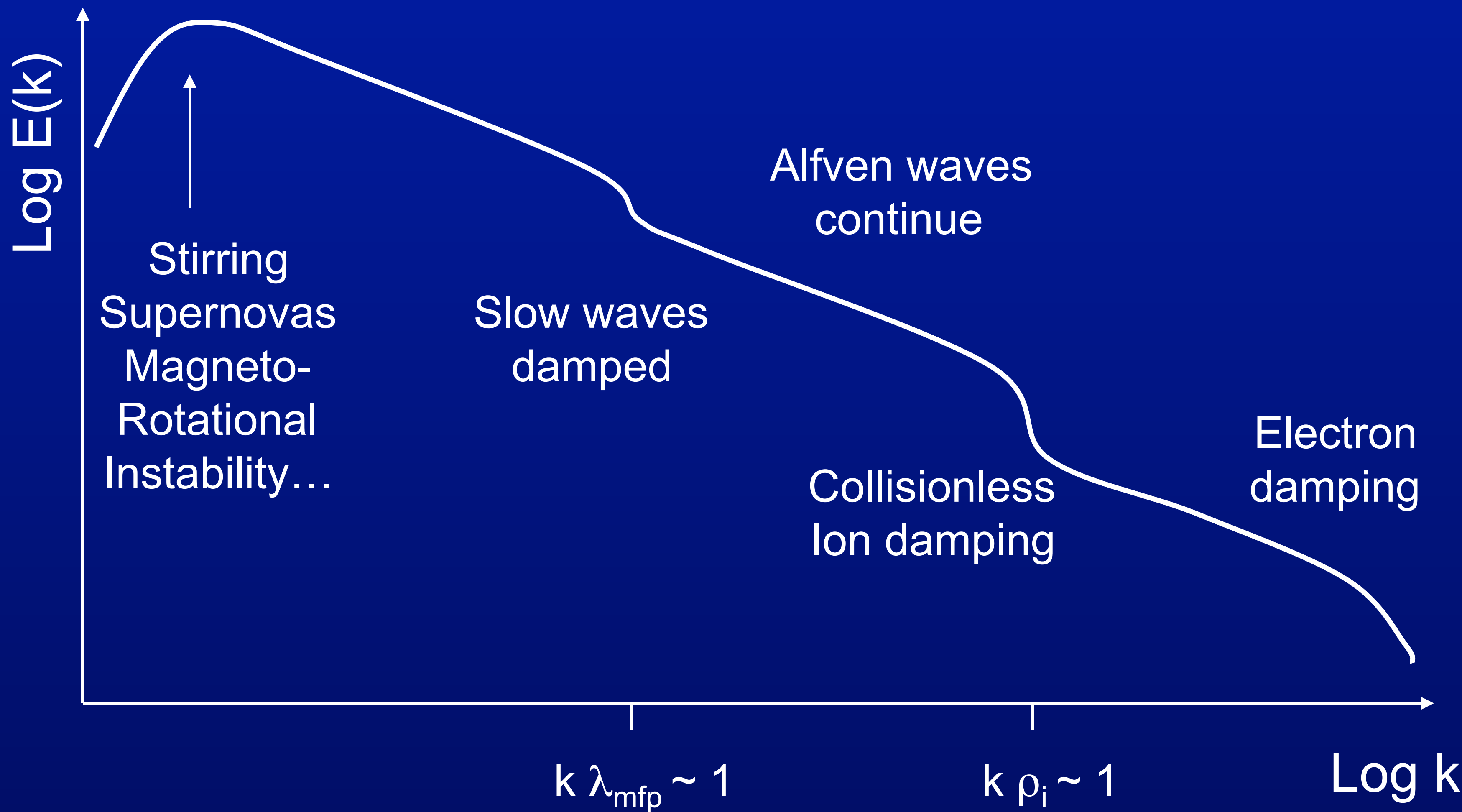
$L_{obs} \sim 10^{-5} \times (0.1\ \dot{M}c^2)$
Why low luminosity? low $\dot{M}$ or low radiative efficiency

Collisionless, magnetized plasma at
R ~ Bondi radius; $\rho_i \ll H$, $\ell_{mfp} \gg H$

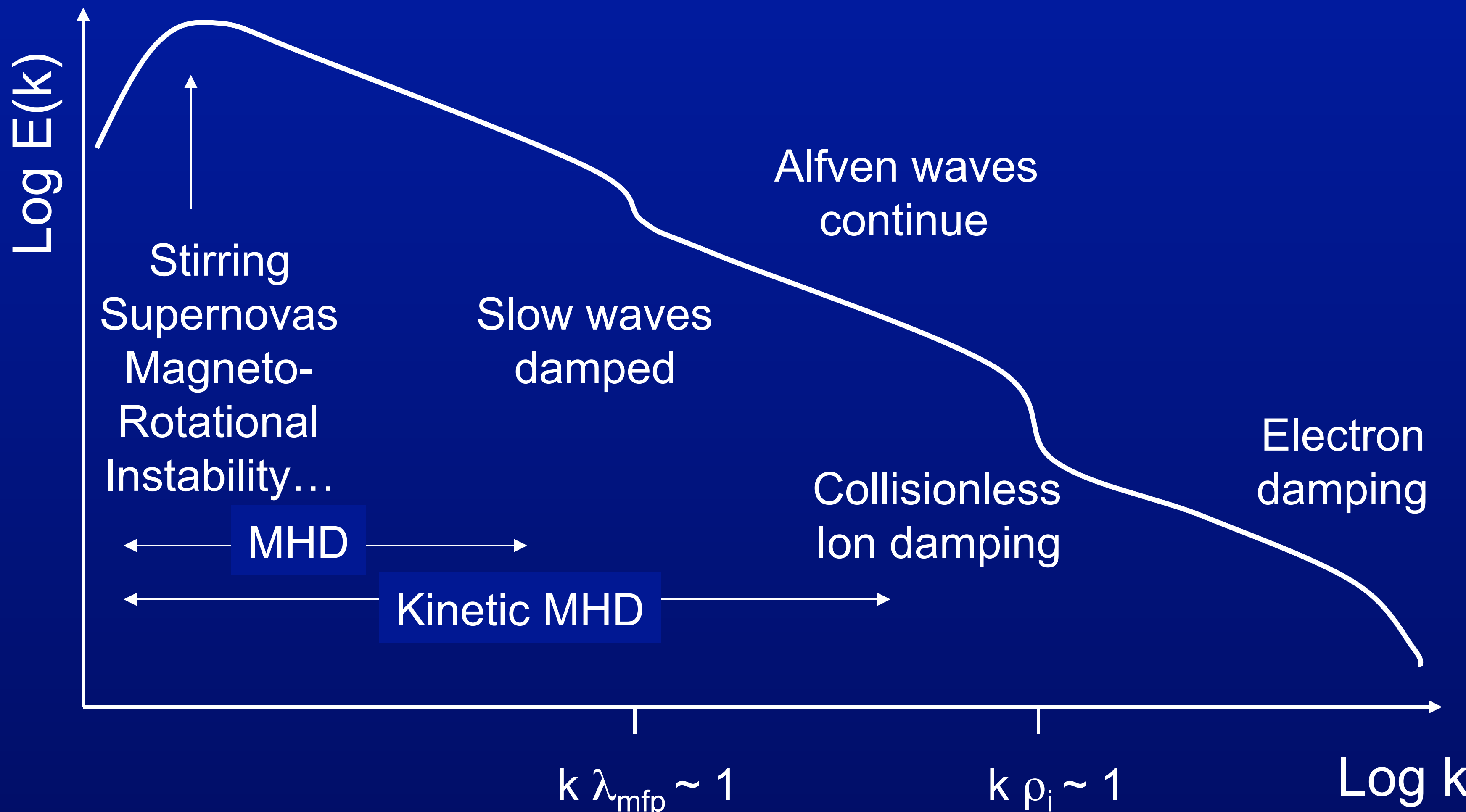


Schödel et al., 2002, A.M. Ghez et al. 2003 http://www.astro.ucla.edu/~ghezgroup/gc

Idealized Problem: What happens to tail of Alfven wave turbulent cascade: e vs i heating?



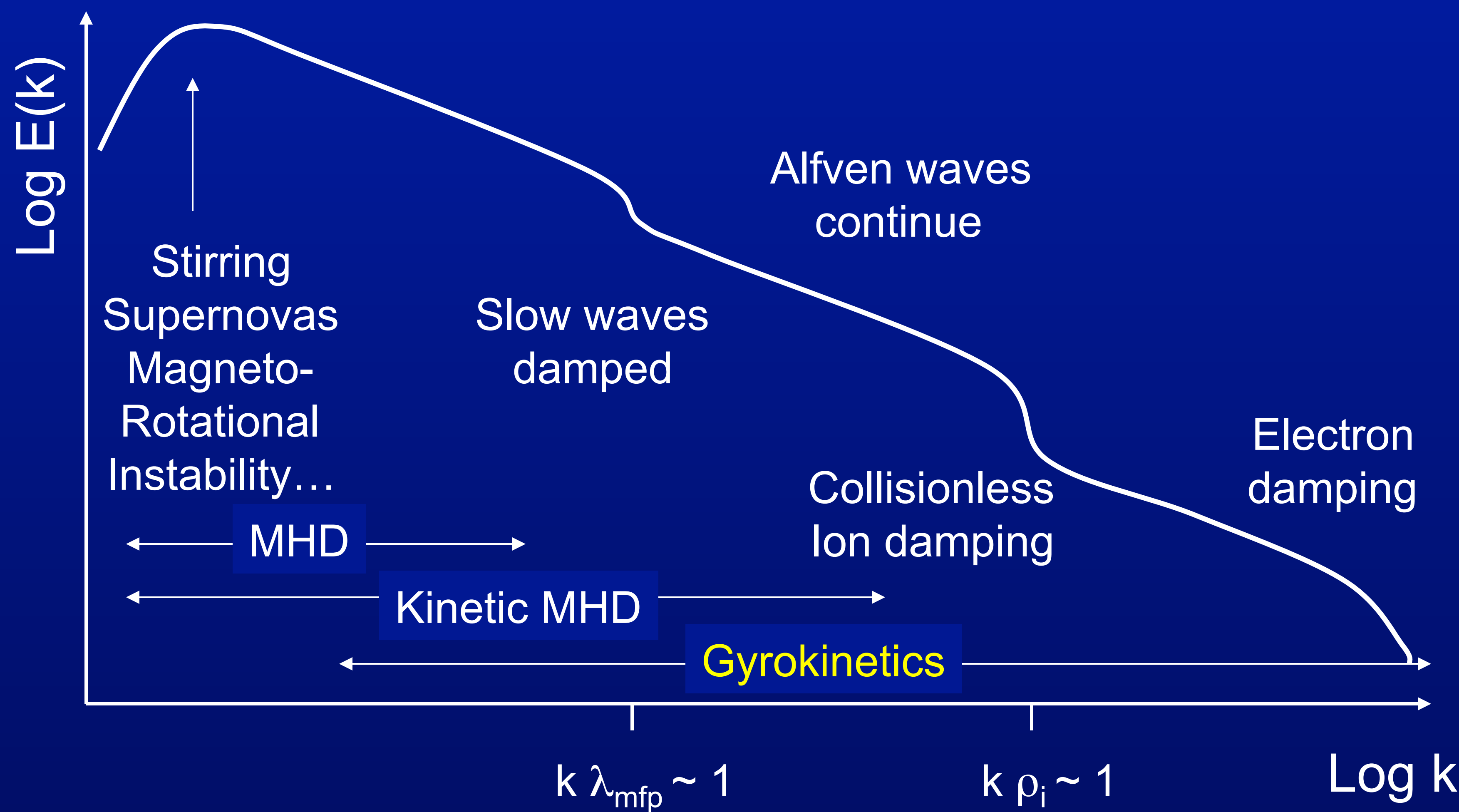
Idealized Problem: What happens to tail of Alfven wave turbulent cascade: e vs i heating?



Kulsrud's formulation of kinetic MHD: anisotropic $P_\perp$ & $P_{||}$, determined by solving drift kinetic equation for distribution function. $(\omega/\Omega_{ci} \sim k_\perp \rho_i << 1)$

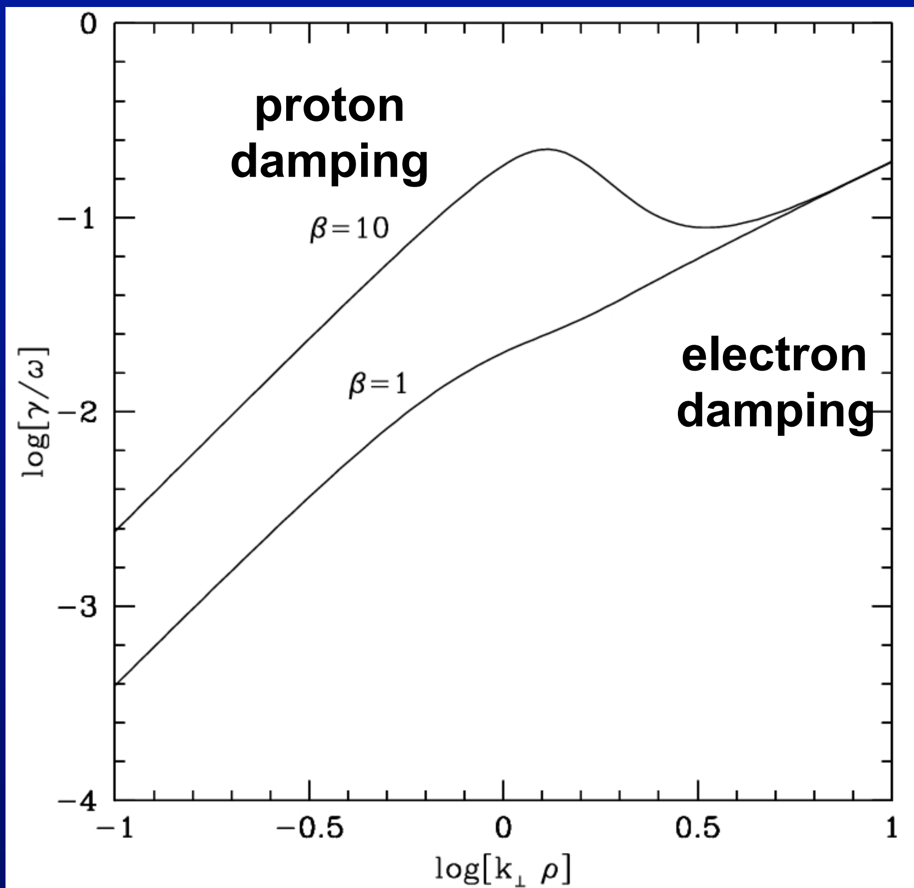
(Varenna 62, Handbook Plasma Physics 83)

Idealized Problem: What happens to tail of Alfven wave turbulent cascade: e vs i heating?



Answer requires more than MHD: collisionless kinetics, finite gyroradius. This is the regime of nonlinear gyrokinetic equations and codes developed in fusion energy research in 1980's and 1990's.

Collisionless Damping on ~ ρ_i scales



Quataert 1998

Linear collisionless damping of Alfven waves with $k_\perp \gg k_{||}$

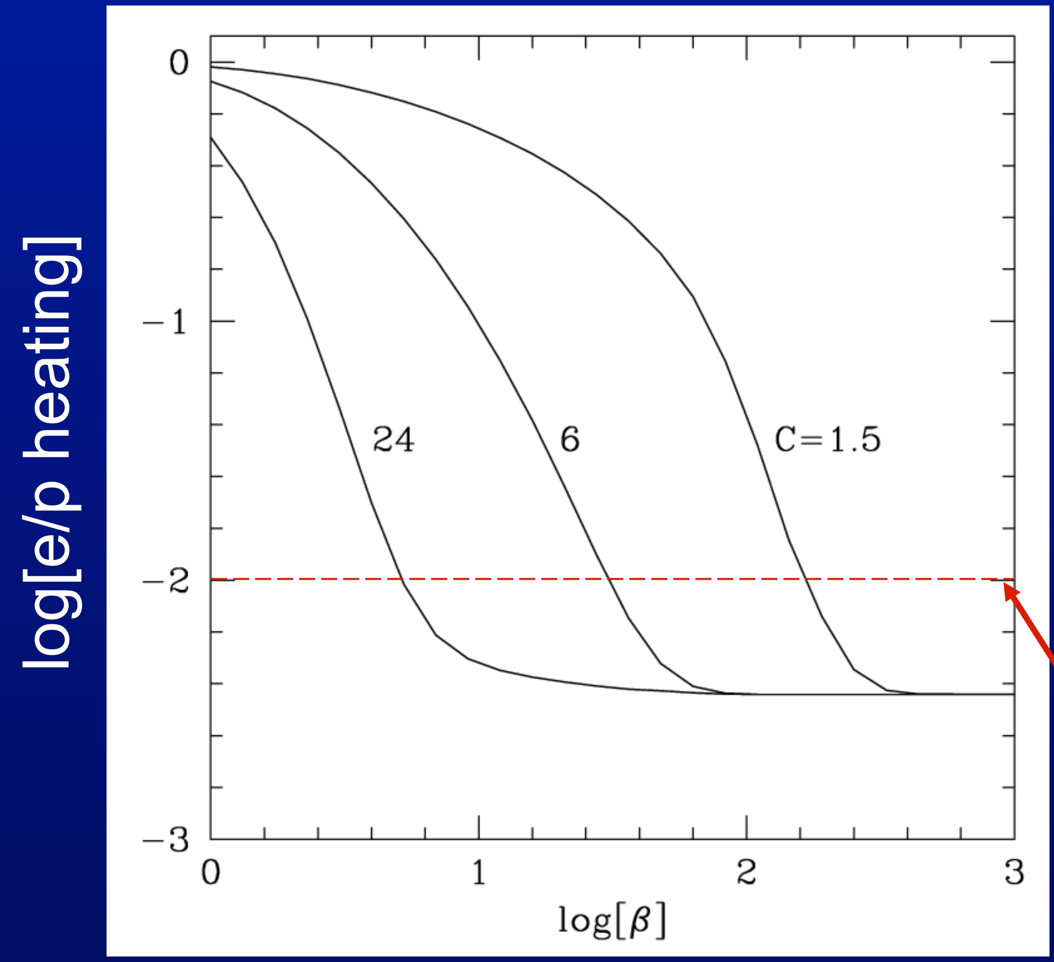
Strong damping requires
γ **~ nonlinear freq. ~** ω

Damping sets inner scale & ⇒ Heating of Plasma

Resulting heating of ions / electrons uncertain: depends on rate turbulence nonlinearly cascades through $k_\perp \rho_p \sim 1$, but this is outside MHD regime ⇒ need Gyrokinetics!

see gyrokinetic simulations with fusion codes (GS2 & GENE) & theory by Howes, TenBarge, Told, et al.

Analytic Estimates of Electron Heating Are Indeterminate



Quataert & Gruzinov 1999

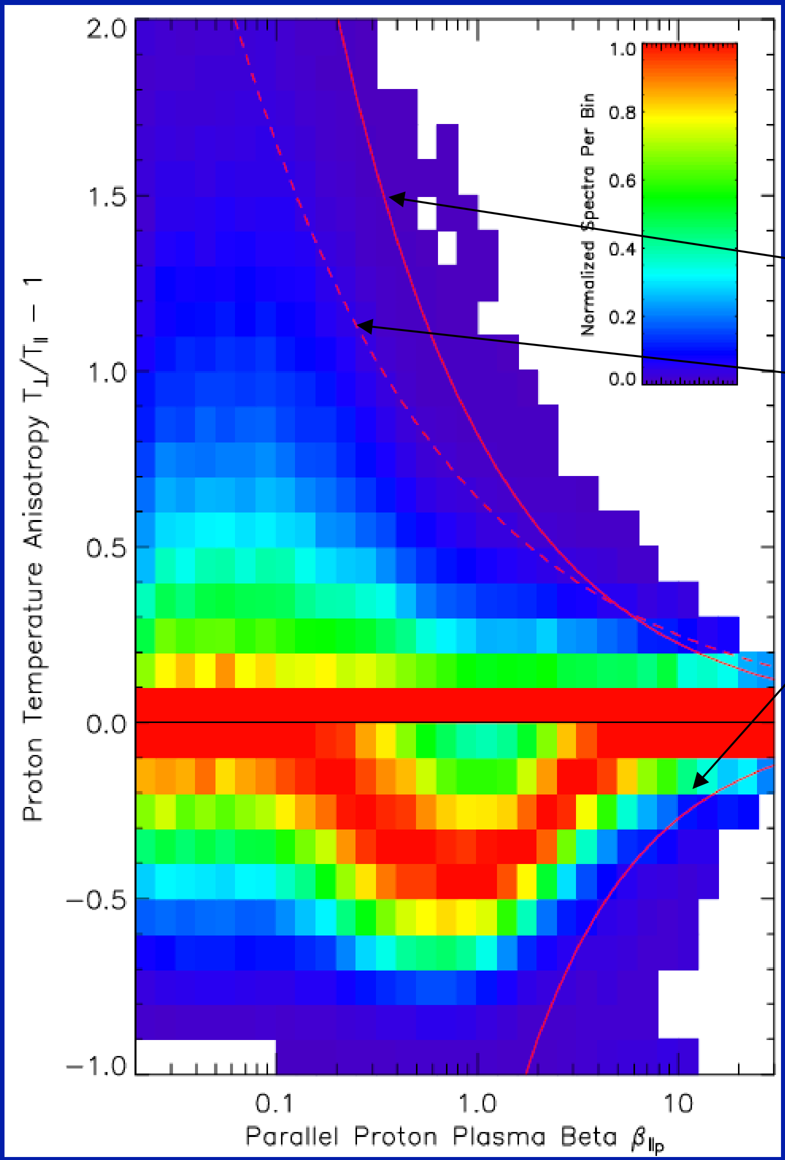
$$C = \frac{T_{\mathrm{Nonlinear}}}{T_{\mathrm{Linear}}}$$

uncertain because damping occurs at $k_\perp \rho_p > 1$ outside MHD regime ⇒ need Gyrokinetic simulations

Low electron heating reqd for ADAF models to explain low luminosity of some black holes

More recent results: see gyrokinetic simulations with fusion codes (GS2 & GENE) & theory by Howes, TenBarge, Told, et al.

Limits on Pressure Anisotropy



$$\left|\frac{p_\perp}{p_\parallel} - 1\right| \le \frac{S}{\beta^\alpha}$$

mirror: S=7, α=1 (to break adiabatic invariance)

ion-cyclotron: S=0.35, α=0.45 for $\gamma/\Omega_i=10^{-4}$

mirror dominates IC for β~10-100

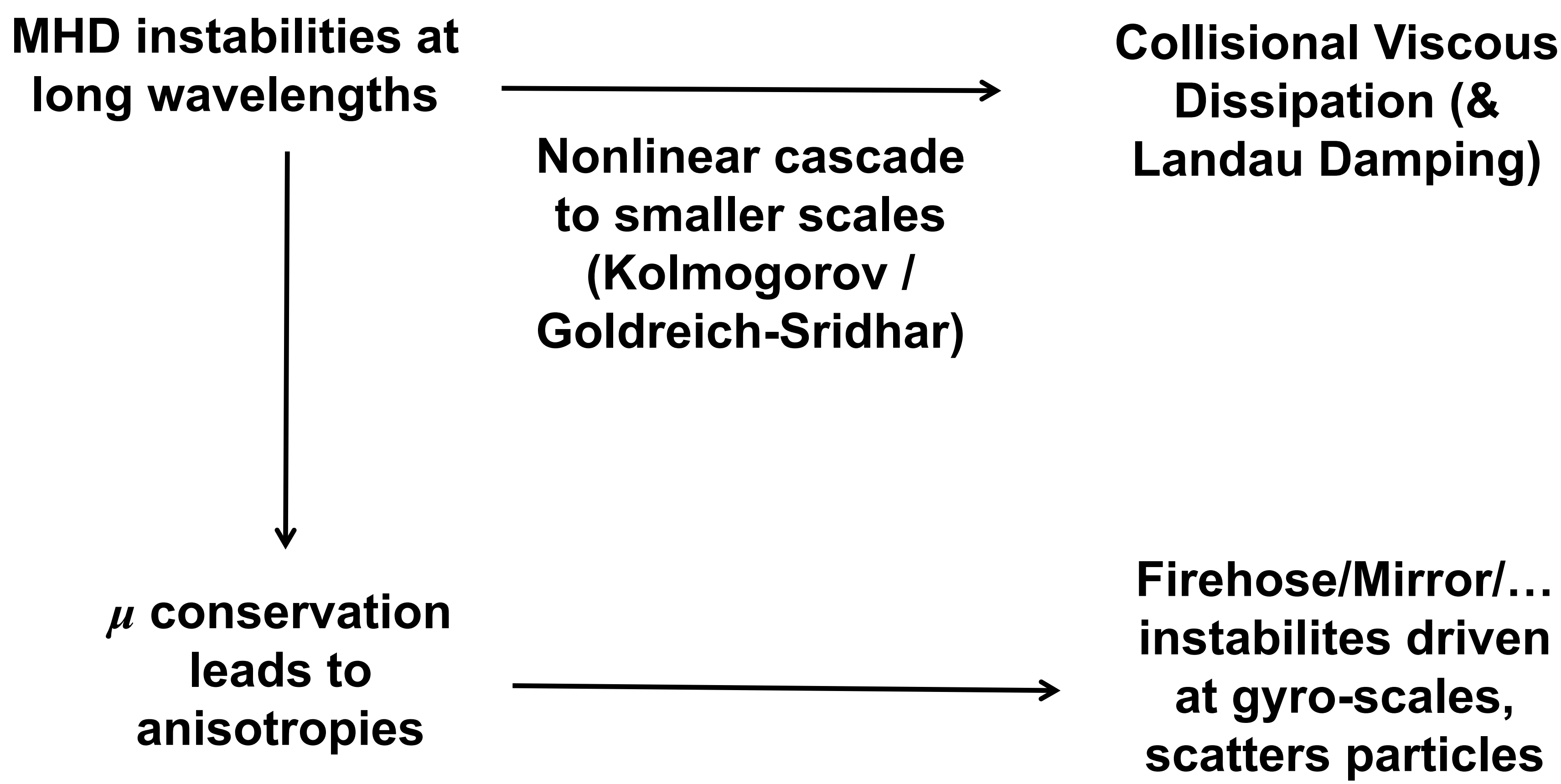
firehose:S>2, α=1

Pressure anisotropy reduced by pitch-angle scattering if anisotropy exceeds threshold.

For electrons with $p_\perp > p_\parallel$ electron whistler instability will isotropize: S=0.13, α= 0.55 (γ/Ω = 5×10^{-8}) *[using WHAMP code]*

[Kasper et al. 2003, Gary & coworkers]

Microinstabilities Driven by Anisotropies Give Alternative Heating Mechanism



Summary:

- Astrophysical plasma turbulence is a rich, fascinating field
- Extreme range of parameters in different types of astrophysical systems, many different types of turbulence and phenomena
- Neutral fluid turbulence has no waves
  MHD turbulence involves linear wave dynamics + nonlinearities
  (Elsasser variables isolates left and right-going waves)
- Critical balance:
  linear parallel dynamics ~ nonlinear perpindicular rates
- Magneto-Rotational Instability: Ubiquitous drive of turbulent viscosity to drive accretion
- Interesting question: as turbulence cascades to small scale, how much electron v. ion heating occurs? need complicated gyrokinetic / kinetic codes. Role of microinstabilities (mirror, firehose, Weibel, ...)?